Another gathering of peculiar interest was the meeting of the Association of American Agricultural Colleges and Experiment Stations, August 11–13, at Berkeley. The name of this association may seem to indicate that it is interesting only to agriculturalists but attention should be called to the fact that the Association includes all of the so-called Land-Grant colleges, which receive aid under various national acts from the United States government. Thus many state universities were represented at this meeting, and such institutions as the Massachusetts Institute of Technology. A subsidiary association of considerable importance to those interested in mathematics was the Land Grant College Engineering Association, which met at the same time. To those who are not aware of the purposes of these two associations it may be pointed out that one very important project before them at present is to secure government aid for the establishment of engineering experiment stations in the various land-grant colleges.

Finally, the meeting of the National Education Association held in Oakland, August 16–28, will be of interest to those who have to do with secondary education either directly or indirectly. No full account of this meeting is possible and we must content ourselves with the statement that it was a very large meeting and that a full report of it will be published by the Association itself. One discussion that occurred there which will be of interest to all who read the MONTHLY was concerned with the establishment of six-year high schools—a movement which would seem to be well under way and to promise great changes and considerable success in the near future.

A large number of other meetings of educational importance were held, including for example the Association of American Universities and the Association of American State Universities.

On the whole the meetings held this summer in California will certainly take rank as one of the most important groups of meetings which have ever been held in the United States.

HISTORY OF MATHEMATICS.

By G. A. MILLER, University of Illinois.

In February, 1640, Descartes wrote as follows: "I am accustomed to distinguish two things in the mathematics, the history and the science. *By the history I mean whatever is already discovered, and is committed to books.* And by the science, the skill of resolving all questions, and thence by investigating by our own industry whatever may be discovered in that science by human ingenuity. He who possesses this faculty has but little need of other assistance, and may therefore be properly called self-sufficient. Now it is much to be wished that this mathematical history, which lies scattered through many volumes, and is not yet entire and complete, were to be all collected into one book."[1]

[1] *The Philosophical Transactions of the Royal Society of London* (Abridged), Vol. 2, 1809, p. 533.

Descartes's definition of history of mathematics, implied by the sentence of the preceding quotation which was italicized by the present writer, may at first surprise many of those who have acquired their idea of the meaning of the term history of mathematics from the modern books bearing this title. A second thought will doubtless reveal difficulties in the way of defining clearly what we really mean by this term. Are the mathematical developments of the first decade and a half of the present century a part of the history of mathematics? If this is the case, should we not begin the study of the history of mathematics by the study of this period?

To direct attention to the feasibility of beginning a study of the history of mathematics near its border which is closest to us, it may be interesting to recall a few mathematical activities which were inaugurated since the beginning of the present century. The one which is probably of most interest to the average teacher is the work done under the general direction of the *International Commission on the Teaching of Mathematics*, which was created during the fourth international congress held at Rome in April, 1908. At first it was intended that this commission should confine its work to secondary mathematics, but it soon appeared desirable to include all mathematical instruction in the scope of its investigators.[1] The central committee is now composed of seven men representing seven leading countries as follows: F. Klein, Germany; G. Greenhill, England; D. E. Smith, United States; H. Fehr, Switzerland; G. Castelnuovo, Italy; E. Czuber, Austria; and J. Hadamard, France.

The magnitude of the work accomplished during the first six years of the existence of this Commission is partly exhibited by the fact that over ten thousand pages of reports prepared under its general direction were issued during this period, according to a statement made at the meeting held at Paris in April, 1914. These reports relate mainly to the materials and methods of teaching mathematics in sixteen different countries, including all those which lead in scientific activities, and they constitute a most valuable addition to the literature on the development of mathematics in various modern countries. Several additional reports have appeared since the given meeting, and it seems probable that the work inaugurated by this Commission will bear fruit for a long time to come.

Another large mathematical undertaking inaugurated during the first decade and a half of the present century is the publication of the great mathematical encyclopedia, entitled *Encyclopédie des Sciences Mathématiques*, the first part of which was published in 1904. More than thirty large volumes of this great encyclopedia have been planned and over five thousand pages have already been published. Unfortunately the great European war has greatly delayed this publication, which promises to become the largest and most valuable mathematical encyclopedia that has ever been written.

A considerable number of mathematical periodicals have been started during the period under consideration. About the beginning of this period the *Trans-*

[1] J. W. A. Young gave an account of the work of this Commission during its first four years, in this MONTHLY, Vol. 19 (1912), p. 161.

actions of the American Mathematical Society was started and it has enjoyed the support of leading American mathematicians from the start. The high standards and careful editorship of this periodical have been of the greatest value in the development of research activity of a high order in our midst. Quite recently an important general scientific journal, devoting considerable attention to mathematics, was started in our country, viz., the *Proceedings of the National Academy of Sciences*. The first number appeared in January of the present year, and the periodical at once received the support of leading investigators in the various scientific fields.

In May, 1911, there was started a new Spanish mathematical journal, entitled *Revista de la Sociedad Matematica Española*, which is of special interest to Americans in view of the fact that the Spanish language is used in large parts of our continent. This periodical is the official organ of the national mathematical society of Spain, which was organized about a month before the periodical was started. It is to be hoped that this society and its journal will do much to organize the mathematical work among the people using the Spanish language, especially since these people have not taken an active part in the development of mathematics during recent centuries, having remained far behind the Italians in this regard.

Another important mathematical periodical entitled *Tôhoku Mathematical Journal* was started in 1911 at Sendai, Japan. It invited from the beginning contributions in English, French, German, Italian, and Japanese, but most of its articles thus far have been in English. This is the first journal devoted mainly to modern advanced mathematics which has been published in Japan, and its international character should do much to advance the interests of higher mathematics in that country.

In view of the fact that Asia took practically no part in the development of mathematics in modern times up to the beginning of the twentieth century, it is of interest to note another thriving mathematical journal started in Asia during the period under consideration. This periodical is entitled *The Journal of the Indian Mathematical Society*, and was started at Madras, India, in February, 1909. The fact that it is the official organ of a society founded in 1907 for the advancement of mathematical study and research in India makes it the more interesting and increases its opportunities for usefulness. Another Indian mathematical periodical was started in 1909, under the title *Bulletin of the Calcutta Mathematical Society*, but only four numbers of this journal, which was to be a quarterly, have been issued thus far.

One of the most noteworthy features of the period under consideration is the rapidly increasing interest in questions relating to the teaching of mathematics, and the great success of the International Commission on the Teaching of Mathematics was doubtless largely due to the fact that the time was ripe for vigorous advances along this line. Reforms of various kinds and of far-reaching significance have received widespread attention. A considerable number of new journals devoted mainly to methods of teaching were established during the

decade and a half under consideration. In our own country *School Science and Mathematics* and *The Mathematics Teacher* may serve as illustrations. The former of these is a continuation of *School Science*, the first number of which appeared in March, 1901; while the latter began to appear as a quarterly in September, 1908.

Perhaps no other undertaking started during the period under consideration exhibits the spirit of this period so clearly as the commencement of the publication of the collected works of the great Swiss mathematician, Leonard Euler. Various efforts had been made earlier to publish these very extensive works but these efforts failed on account of the large amount of money required for the publication. In September, 1909, the Swiss Society of Naturalists, having received through national and international subscriptions and through donations about one hundred thousand dollars for this purpose, announced that it would undertake this great publication. Two years later the first volume was published, and several other volumes followed in rapid succession, but it soon appeared that, in view of the discovery of new MSS., the entire publication would cost nearly twice as much as the original estimate, and that it would fill more than forty-five large volumes. This led to the formation of a unique international mathematical society, called the *Leonard Euler-Gesellschaft*, whose main object is to aid this publication.

The developments which have been noted are of a general nature and they constitute merely evidences of the fact that there was real mathematical growth during the period under consideration. It would be of more interest to consider some of the mathematical advances themselves, but these are too numerous and too extensive to be described in a brief article. If it is observed that the *Jahrbuch über die Fortschritte der Mathematik* fills annually a volume of over a thousand pages in giving titles and brief reviews of the new literature on mathematics, it results that for the decade and a half under consideration it would require more than fifteen thousand pages to present even such a limited consideration of the mathematical progress as is contained in the given review. Hence it is clear that a history of the development of mathematics during the first decade and a half of the twentieth century might well fill many volumes.

The main object of the present article is to raise the question whether such books on the history of mathematics as Ball, Cajori, Cantor, etc., are not apt to convey a very incorrect notion of what the history of mathematics really is, and of what developments should be embodied in a first course on this subject. In fact, Cantor's *Vorlesungen über Geschichte der Mathematik* are confined to the developments which preceded the nineteenth century, and hence they do not touch the period in which most of our present mathematical literature originated. Possibly some of the other books could be used to the best advantage as text-books by beginning near the end and working forward, but a really inspiring course in the history of mathematics would probably have to be based largely upon the literature contained in the recent journals.

A course in the history of mathematics should not tend to create a veneration

of the past at the expense of an appreciation of the present. It would be better to be ignorant of the Pythagorean school than to be ignorant of the modern mathematical schools. It would be better to be ignorant of the Greek Eleatic school than to be ignorant of some of the fundamental results in the modern theory of aggregates. It would be better to be able to name with some intelligence ten of the most eminent living mathematicians than to be able to name that number of ancient Arabians who helped to preserve and to transmit the mathematical lore of the ancient Greeks.

One difficulty about beginning a course in the history of mathematics with the developments of the last ten or fifteen years is that the new theorems and theories have not yet established their true value in the permanent literature. Mathematical fashions are changeable, but mathematical worth is permanent. Recent mathematical history is apt to be affected by the fashions while the older mathematical history is based upon established permanent values. These objections do not apply so strongly to a study of the various present activities which were inaugurated with a view to furthering mathematical interests, and these activities should be well understood by all those who teach mathematics.

It is also true that it is somewhat more difficult to keep in touch with the recent developments than it is to study once for all some accounts of the older developments. This difficulty has been greatly reduced in recent years by the increase in the periodical literature and by the various aids to make rapid surveys of the main advances in various fields. There is less and less excuse for living entirely in the mathematical past. The growing dynamic elements of mathematics naturally appeal to a large number, especially to the younger people, and these elements are best understood if they are observed in their natural surroundings and in real life.

The given quotation from Descartes seems to imply that he thought that all the mathematical history of his day could have been collected into *one* book. We noted above that the large French mathematical encyclopedia is expected to fill more than thirty volumes, and this will not include all the known mathematical results. If it is observed that more than two thousand original mathematical articles are published every year, it is clear that mathematical history is growing more rapidly than one man could write it. Hence such a thing as a complete mathematical history seems out of question. It is, however desirable to know something about this history, and especially about that part which lies closest to us.

It is of interest to inquire what distinguishes the history of mathematics from other mathematical writings, in case we assume that there is a difference, as is commonly done. According to Descartes's view practically all the literature which appears in our better journals of mathematics should be regarded as historical, while many others would probably be inclined to contend that only a small part of this literature should be classed with history. All might agree on the statement that every mathematical advance is making mathematical history, but some would probably hold that this advance would become history through

some process of maturing, while the process itself might be vague and hence undefinable. Judging from the writings which are now commonly classed under mathematical history it would appear that the chronological element was generally considered an essential element in a direct historical paper. In an indirect historical paper,—for instance, one discussing how mathematical history should be written—this element would not need to be present.

Next in importance to the chronological element in the usual historical presentation of a mathematical subject is the human element. Even the mere names of those who have enriched mathematical thought by pointing out logical steps leading to views of unusual beauty or to regions of unusual fruitfulness serve to establish a sense of comradeship. This sense is intensified by more or less complete biographical notices. It should, however, be observed that these names and these biographical sketches serve other useful purposes. The names of great mathematicians may be used to unify varied mathematical results, while the biographical sketches serve to unify mathematical knowledge and knowledge relating to other lines of thought.

While these considerations may throw some light on the term history of mathematics they are not intended to convey the idea that this term could be defined in a perfectly satisfactory manner. Like some other useful mathematical terms, the term history of mathematics will probably always remain without a real definition. While one may not know where mathematical history begins, yet there are some writings which all agree to classify under history and others which few would classify under this heading. On the other hand, there seems to be a strong tendency towards increasing the historical element in modern mathematical writings so that the writings which are zero per cent. history are becoming less and less common. It is to be hoped that the mathematical teaching which devotes zero per cent. of the time to history, and the mathematical history which includes zero per cent. of the present-day mathematical activities will also become less and less common.

ON THE CIRCLES OF APOLLONIUS.

(Concluded)

By NATHAN ALTSHILLER, University of Colorado.

(By some unaccountable oversight this last page of DR. ALTSHILLER'S article was omitted from the October issue. EDITORS.)

12. The three pairs of points A, A'; B, B'; C, C' of ω are perspective from K; they belong therefore to the same involution on ω, K and l being respectively the pole and the axis of the involution[1] (9A). Hence the couples of lines AB, $A'B'$; BC, $B'C'$; AC, $A'C'$ meet on l. Now, C_3, C_1, C_2 are the points common to l and the lines AB, BC, AC respectively. Hence:

[1] Russell, *loc. cit.*, pp. 217, 218.